Int J Theor Phys (2011) 50: 27–34
DOI 10.1007/s10773-010-0489-9

Cosmological Models with Variable G in C-Field Cosmology

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Received: 10 July 2010 / Accepted: 7 September 2010 / Published online: 22 September 2010

Abstract Cosmological models with variable G in C-field cosmology for barotropic fluid distribution in FRW space-time are investigated. To get the deterministic model of the universe, we have assumed that $G = R^n$ where R is the scale factor and n the constant. To obtain the results in terms of cosmic time t, we have assumed $n = -1$. We find that for $n = -1$, Creation field (C) and spatial volume increase with time, G and ρ (matter density) decreases with time, the model represent accelerating universe. Thus inflationary scenario exists in the model. The model is also free from horizon. The results so obtained match with the astronomical observations.

Keywords C-field cosmology · Variable G

1 Introduction

The importance of gravitation on large scale is due to the short range of the strong and weak forces and to the fact that electromagnetic force becomes weak because of the global neutrality of matter as pointed by Dicke and Peebles [1]. Dicke [2] stressed that the Earth is such a complex system that it would be difficult to use it as a source of evidence for or against the existence of time variation of the gravitational constant. Motivated by the occurrence of large numbers hypothesis, Dirac [3] proposed a theory with a variable gravitational constant. Pochoda and Schwarzschild [4], Ezer and Cameron [5] and Gamow [6] studied the solar evolution in presence of a time varying gravitational constant. They arrived on the conclusion that under Dirac hypothesis, the original nuclear resources of the Sun would have been burned by now. This results from the fact that an increase of the gravitational constant is equivalent to an increase of the star density (because of Poisson equation). Demarque et al. [7] considered an ansatz in which $G \propto t^{-n}$ and showed that $|n| < 0.1$ corresponds to

$$\left|\frac{\dot{G}}{G}\right| < 2 \times 10^{-11}\ \mathrm{yr}^{-1}$$

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Gaztanaga et al. [8] considered the effect of a variation of the gravitational constant on the cooling of white dwarfs and on their luminosity function and concludes that

$$\left|\frac{\dot{G}}{G}\right| < 3 \times 10^{-11}\ \mathrm{yr}^{-1}$$

Barrow [9] assumed that $G \propto t^{-n}$ and obtained from helium abundances for $-5.9 \times 10^{-3} < n < 7 \times 10^{-3}$,

$$\left|\frac{\dot{G}}{G}\right| < (2 \pm 9.3)\ \mathrm{h} \times 10^{-12}\ \mathrm{yr}^{-1}$$

by assuming a flat universe.

Subsequently, mathematically well posed alternative theories of gravity were developed to generalize Einstein's general theory of relativity by including variable G and satisfying conservation equation. To achieve possible unification of gravitation and elementary particle physics or to incorporate Mach's principle in general relativity, many attempts have been proposed for the possible extension of general relativity with time dependent G [10–13].

In the early universe, all the investigations dealing with the physical processes use a model of the universe, usually called the 'big-bang model'. However, the big-bang model is known to have short comings in the following aspects: (i) The model has singularity in the past and possibly one in future. The singularity signals mathematical inconsistency and physical incompleteness. (ii) The conservation of energy is violated in the big-bang model. Since the left-hand side of Einstein's field equation has zero divergence, on the other hand, the energy density in the big-bang model is positive definite. Thus, it is impossible for matter to come into existence without violating energy conservation. (iii) The big-bang models based on reasonable equations of state lead to a very small particle horizon in the early epochs of the universe. This fact give rise to the 'horizon problem' in the universe. (iv) No consistent scenario exists within the frame work of big-bang model that explains the origin, evolution and characteristic of structures in the universe at small scales. (v) Flatness problem.

C-field (a negative-energy field) has solved the problem of horizon and flatness faced by the big-bang model. These problems in standard big-bang scenario have been solved by inflation. The problems of flatness and isotropy of the universe have been solved by inflationary theory within the big-bang (Guth [14]). Inflation has also given a basis for understanding the origin of large structures arising from quantum fluctuations in inflationary period as discussed by many authors viz. Linde [15], Grön [16], Barrow [17], Rothman and Ellis [18], Madsen and Coles [19], Linde [20], Bali and Jain [21], Chervon [22], Reddy and Naidu [23].

If a model successfully explains creation of positive-energy matter without violating the conservation of energy then it is necessary to have some degree of freedom which acts as a negative energy mode. All quantum gravitational models which describe the creation consistently [14] use such a 'negative energy mode' arising from the scale degree of freedom of gravity. Thus a negative-energy field provides a natural way for creation of matter. We know that the classical singularity theorems cease to be operational when positivity of energy density is not guaranteed. Hoyle and Narliker [24] adopted a field theoretic approach introducing a massless and chargeless scalar field to account for creation of matter. In C-field theory, there is no big-bang type singularity as in the steady-state theory of Bondi and Gold [25]. Narlikar [26] has pointed out that matter creation is accomplished at the expense of negative energy C-field. Narlikar and Padmanabhan [27] have investigated a solution of

Einstein's field equations which admits radiation and a negative energy massless scalar creation field as a source. They have shown that cosmological model based on this solution satisfies all the observational tests and is a viable alternative to the standard big-bang model and free from singularity and particle horizon and also provides a natural explanation to the flatness problem. Also the phantom field is the revival of C-field as discussed by many authors viz. Caldwell [28], Gibbons [29], Singh et al. [30], Giocomini and Lara [31], Paul and Paul [32]. Vishwakarma and Narlikar [33] have discussed modeling repulsive gravity with creation. Recently Bali and Tikekar [34], Bali and Kumawat [35] have investigated C-field cosmological models for dust distribution in FRW space-time with variable gravitational constant.

In this paper, we have investigated C-field cosmological models for barotropic perfect fluid distribution with variable G in the frame work of FRW space-time. To get the deterministic solution, we have assumed $G = R^n$ where R is the scale factor and n is a constant. We have also discussed a cosmological model in terms of cosmic time for particular value of $n = -1$. In this model inflationary scenario exists and horizon does not exist. The physical aspects related with the astronomical observations are also discussed.

2 The Cosmological Models

We consider the homogeneous and isotropic cosmological model described by Robertson-Walker line-element:

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{1}$$

where $k = 0, \pm 1$.

Einstein's field equation by introduction of C-field is modified by Hoyle and Narlikar [11] as

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G[\underset{(m)}{T}{}_i^j + \underset{(c)}{T}{}_i^j] \tag{2}$$

The energy momentum tensor for perfect fluid is taken as

$$\underset{(m)}{T}{}_i^j = (\rho + p)v_i v^j - p g_i^j \tag{3}$$

and

$$\underset{(c)}{T}{}_i^j = -f\left(C_i C^j - \frac{1}{2}g_i^j C^\alpha C_\alpha\right) \tag{4}$$

where $f > 0$ is the coupling constant between matter and creation field, and $C_i = \frac{dC}{dx^i}$. The field equations (2) for the metric (1) lead to

$$\frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} = 8\pi G(t)\left[\rho - \frac{1}{2}f\dot{C}^2\right] \tag{5}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 8\pi G(t)\left[\frac{1}{2}f\dot{C}^2 - p\right] \tag{6}$$

The conservation equation

$$[8\pi G T_i^j]_{;j} = 0 \tag{7}$$

leads to

$$8\pi \dot{G}\left[\rho - \frac{1}{2} f \dot{C}^2\right] + 8\pi G\left[\dot{\rho} - f \dot{C}\ddot{C} + 3\rho\frac{\dot{R}}{R} - 3 f \dot{C}^2 \frac{\dot{R}}{R} + 3p\frac{\dot{R}}{R}\right] = 0 \tag{8}$$

Following Hoyle and Narlikar [11], the source equation of C-field: $C^i_{;i} = 0$ leads to $C = t$ for large r. Thus $\dot{C} = 1$.

Now using $\dot{C} = 1$, (5) leads to

$$8\pi G\rho = \frac{3\dot{R}^2}{R^2} + \frac{3k}{R^2} + 4\pi G f \tag{9}$$

Using $\dot{C} = 1$ and barotropic condition $p = \gamma\rho$ in (6), we have

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = 4\pi G f - 8\pi G\gamma\rho \tag{10}$$

where $0 \leq \gamma \leq 1$.

Equations (9) and (10) lead to

$$\frac{2\ddot{R}}{R} + (1+3\gamma)\frac{\dot{R}^2}{R^2} = (1-\gamma)4\pi G f - (1+3\gamma)\frac{k}{R^2} \tag{11}$$

To obtain the deterministic solution, we assume

$$G = R^n \tag{12}$$

where n is a constant and R is the scale factor.

Equations (11) and (12) lead to

$$2\ddot{R} + (1+3\gamma)\frac{\dot{R}^2}{R} = (1-\gamma)4\pi f R^{n+1} - \frac{k}{R}(1+3\gamma) \tag{13}$$

Let us assume $\dot{R} = F(R)$.

This leads to $\ddot{R} = FF'$ with $F' = \frac{dF}{dR}$. Thus (13) leads to

$$\frac{dF^2}{dR} + \frac{(1+3\gamma)}{R}F^2 = (1-\gamma)4\pi f R^{n+1} - \frac{k}{R}(1+3\gamma) \tag{14}$$

which leads to

$$F^2 = \frac{4\pi f(1-\gamma)R^{n+2}}{n+3\gamma+3} - k \tag{15}$$

The integration constant has been taken zero for simplicity. The equation (15) leads to

$$\frac{dR}{\sqrt{4\pi f(1-\gamma)R^{n+2} - k(n+3\gamma+3)}} = \frac{dt}{\sqrt{n+3\gamma+3}} \tag{16}$$

To obtain the determinate value of R in terms of cosmic time t, we consider

$$n = -1$$

Putting $n=-1$ in (16), we have

$$\frac{dR}{\sqrt{R-\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}}}=\sqrt{\frac{4\pi f(1-\gamma)}{3\gamma+2}}dt \tag{17}$$

Equation (17) leads to

$$R=(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)} \tag{18}$$

where

$$a=\frac{1}{2}\sqrt{\frac{4\pi f(1-\gamma)}{(3\gamma+2)}} \tag{19}$$

$$b=\frac{N}{2} \tag{20}$$

and N is the constant of integration. Thus we have

$$G=R^{-1}=\left[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}\right]^{-1} \tag{21}$$

From (9), (18) and (21), we have

$$8\pi\rho=\frac{12a^2(at+b)^2+3k}{[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]}+4\pi f \tag{22}$$

Thus the metric (1) after using (18) leads to

$$ds^2=dt^2-\left[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}\right]^2\left[\frac{dr^2}{1-kr^2}+r^2d\theta^2+r^2\sin^2\theta d\phi^2\right] \tag{23}$$

Now using $p=\gamma\rho$, (8) leads to

$$8\pi[G\dot{\rho}+\dot{G}\rho]-4\pi\dot{G}f\dot{C}^2-8\pi Gf\dot{C}\ddot{C}-24\pi G\frac{\dot{R}}{R}f\dot{C}^2+24\pi G\rho\frac{\dot{R}}{R}(1+\gamma)=0 \tag{24}$$

Substituting (18), (21) and (2) into (24) yields

$$\begin{aligned}&\frac{d\dot{C}^2}{dt}+\frac{10a(at+b)}{[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]}\dot{C}^2\\ &\quad=\frac{2\pi fa(at+b)(3\gamma-2)}{\pi f[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]}\\ &\qquad+\left[\frac{6a^3(at+b)^3(3\gamma+2)+\frac{3}{2}ka(at+b)(3\gamma+1)+\frac{6a^3k(3\gamma+2)(at+b)}{4\pi f(1-\gamma)}}{\pi f[(at+b)^2+\frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]^2}\right]\end{aligned} \tag{25}$$

To reach the deterministic value of $\dot{C}$, we assume $a=1$, $b=0$.

Thus (25) leads to

$$\frac{d\dot{C}^2}{dt} + \frac{10t}{[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]}$$
$$= \frac{2\pi f t(3\gamma+2)}{\pi f[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]}$$
$$+ \left[\frac{6t^3(3\gamma+2) + \frac{3}{2}kt(3\gamma+1) + \frac{6k(3\gamma+2)t}{4\pi f(1-\gamma)}}{\pi f[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}]^2}\right] \tag{26}$$

Equation (26) leads to

$$\dot{C}^2\left[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}\right]^5 = \frac{1}{\pi f}\int\left[6t^3(3\gamma+2) + \left(\frac{3}{2}k(3\gamma+2) + \frac{6k(3\gamma+2)}{4\pi f(1-\gamma)}\right)t\right]$$
$$\times\left[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}\right]^3 dt$$
$$+ \int 2t(3\gamma+2)\left[t^2 + \frac{k(3\gamma+2)}{4\pi f(1-\gamma)}\right]^4 dt \tag{27}$$

From (27), we have

$$\dot{C}^2 = \frac{1}{\pi f}\left(\frac{3\gamma+2}{1-\gamma}\right) \tag{28}$$

Thus, we have

$$\dot{C} = \sqrt{\frac{3\gamma+2}{\pi f(1-\gamma)}} \tag{29}$$

which leads to

$$C = \sqrt{\frac{(3\gamma+2)}{\pi f(1-\gamma)}}t \tag{30}$$

Taking $\pi f = (\frac{3\gamma+2}{1-\gamma})$, we find $\dot{C} = 1$, which agrees with the value used in the source equation. Thus creation field C is proportional to time t and the metric (1) for constraints mentioned above, leads to

$$ds^2 = dt^2 - \left[t^2 + \frac{k}{4}\right]^2\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] \tag{31}$$

The homogeneous mass density ρ, the gravitational constant G, the scale factor R and the deceleration parameter q for the model (31) are given by

$$8\pi\rho = \frac{12t^2+3k}{t^2+k} + \frac{4(3g+2)}{1-\gamma} \tag{32}$$

$$G = \left(t^2 + \frac{k}{4}\right)^{-1} \tag{33}$$

$$R = \left(t^2 + \frac{k}{4}\right) \tag{34}$$

$$q = -\left(\frac{2t^2 + \frac{k}{2}}{4t^2}\right) \tag{35}$$

3 Conclusions

For the model (31), the matter density (ρ) decreases with time. The scale factor (R) increases with time. $|\frac{\dot{G}}{G}| \simeq \frac{1}{t} = H$. $G \to$ constant when $t \to 0$ and $G \to 0$ when $t \to \infty$. The deceleration parameter (q) < 0 which indicates that the model (31) represents an accelerating universe. Thus inflationary scenario exists in the model (31). The creation field (C) increases with time and $\dot{C} = 1$ which agrees with the value taken in source equation. When $t = 0$ then $\rho =$ constant. This result may be interpreted as: Referring to Narlikar [36], Hawking and Ellis [37], the matter is supposed to move along the geodesic normal to the surface $t =$ constant. As the matter moves further apart, it is assumed that more matter is continuously created to maintain the matter density at constant value. For $k = 0$, $\gamma = 0$ and for $k = \pm 1$, $\gamma = 0$, we obtain the same results as obtained by Bali and Tikekar [34], Bali and Kumawat [35] respectively.

The coordinate distance to the horizon $r_H(t)$ is the maximum distance a null ray could have traveled at time t starting from the infinite past i.e.

$$r_H(t) = \int_{\infty}^{t} \frac{dt}{R(t)}$$

We could extend the proper time t to $(-\infty)$ in the past because of the non-singular nature of the space-time. Now

$$r_H(t) = \int_0^t \frac{dt}{\alpha t} \quad \text{where } \alpha = \sqrt{\frac{4(3\gamma + 2) - k(3\gamma + 1)}{3\gamma + 1}}$$

This integral diverges at lower limit showing that the models (31) is free from horizon.

Acknowledgement The authors are thankful to Professor J.V. Narlikar for useful discussions and suggestions.

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